Forensic Quantification of Portfolio Churning in Options Accounts

Mourad E. Mazouni, PhD, PMP

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ABSTRACT

A litigation-ready framework for detecting excessive trading in options portfolios. The method combines cost-equity ratio analysis, implied volatility extraction, and Monte Carlo profit probability simulation. In the examined account, the cost-equity ratio reached 136%, ten option positions showed aggregate profit probability below 8%, and cumulative fee drag exceeded any feasible market gain. The framework produces evidence packages suitable for FINRA arbitration and federal court proceedings.

Keywords: Forensic Finance · Options · Compliance · Litigation · Monte Carlo

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SECTION 1

Executive Insight

This paper develops a quantitative framework for detecting and measuring excessive trading in options portfolios. The method produces court-admissible evidence by combining three independent tests: cost-equity ratio benchmarking against court-accepted thresholds, option-level break-even probability computation under lognormal dynamics, and Monte Carlo simulation of aggregate portfolio profit probability under the realized fee structure.

In the examined account, the cost-equity ratio reached 136%, far exceeding the threshold of 20% established in Mihara v. Dean Witter and subsequent FINRA arbitration precedents. The aggregate profit probability for the ten-position option portfolio was below 8%, confirming that the fee structure rendered profitable outcomes statistically implausible regardless of market direction.

SECTION 2

Legal Framework and Precedent

Excessive trading claims in the United States rest on three elements, codified in the BGH (Bundesgerichtshof) three indicia framework and adopted by FINRA: (1) the broker controlled the account, (2) trading was excessive relative to stated objectives, and (3) the broker acted with intent or

reckless disregard.

Quantitative evidence addresses the second element. The cost-equity ratio, defined as total commissions divided by average account equity, is the primary metric. Court-accepted thresholds from Mihara v. Dean Witter (1980), Costello v. Oppenheimer (1986), and CFTC v. Savage (1985) place the presumptive churning boundary at 20% for stock accounts and higher for leveraged derivatives accounts. An annualized turnover ratio exceeding 6x is independently indicative.

SECTION3

Cost-Equity Analysis

The cost-equity ratio for the examined account is computed as: CER = Total Commissions / Average Account Equity = EUR 81,200 / EUR 59,700 = 136%. This means the account must generate returns exceeding 136% merely to recover the costs imposed by the broker, before any market return accrues to the client.

The CER of 136% exceeds the Mihara threshold of 20% by a factor of 6.8. Even under the more permissive standards applied to options accounts, where higher commission ratios reflect higher per-trade costs, a CER above 50% is considered presumptively excessive. The 136% figure places this account in the top fraction of a percent of commission-to-equity ratios in comparable accounts.

SECTION4

Option Pricing and Break-Even Analysis

Individual option positions are analyzed using the Black-Scholes framework to determine the market move required for each position to break even after commissions. For a call option purchased at price C with commission k, the break-even condition requires the underlying to exceed S_BE = K + C + k at expiration.

The probability of reaching this break-even level is computed under geometric Brownian motion: dS = μ * S * dt + σ * S * dW. Using the implied volatility observed at trade entry, each position yields a closed-form break-even probability. Across all ten positions in the examined account, the average break-even probability was 18%, with the lowest at 4% for a deep out-of-the-money call with a 9% commission charge.

SECTION5

Profit Probability Extraction

For each option position, the profit probability Pr(profit > 0) is extracted from the observed implied volatility at entry. The second profit probability, Pr2, accounts for the commission-adjusted break-even and is computed in closed form under lognormal terminal price distribution.

The ten positions in the examined account showed implied volatilities ranging from 22% to 58% and profit probabilities ranging from 4% to 31%. The volume-weighted average profit probability was 14.2%. For a rational, cost-sensitive investor, this expected hit rate would require a profit-to-loss ratio exceeding 6:1 on winning trades to achieve a positive expected return, a condition not supported by the strike selection pattern observed.

SECTION6

Monte Carlo Aggregate Framework

Portfolio-level profit probability is estimated via Monte Carlo simulation. Correlated geometric Brownian motion paths are generated for all underlying assets simultaneously, preserving the empirical correlation structure. For each of 100,000 simulated paths, the realized P&L; is computed for every position including all commissions and fees, then aggregated at portfolio level.

The aggregate profit probability, P(portfolio P&L; > 0), is the fraction of simulated paths producing a positive total return. In the examined account, this probability was 7.8%, meaning that under 92.2% of market scenarios, the client loses money net of fees. The 95% confidence interval for terminal portfolio value was [-62%, -14%], indicating structural loss under virtually all market conditions.

SECTION7

Scenario Analysis

Three market scenarios are examined: (1) a trending bull market with the underlying appreciating 15% over the holding period, (2) a flat market with 0% return, and (3) a moderate correction of -10%. Under the bull scenario, the portfolio returns -8% net of fees. Under the flat scenario, -34%. Under the correction, -52%.

The structural dominance of fee drag is evident: even under the most favorable market condition, the account loses money because commissions consumed 136% of equity. No reasonable market outcome can overcome this cost structure. This finding constitutes the quantitative core of the churning evidence package.

SECTION8

Institutional Implications

For compliance departments, the framework provides a systematic screening tool for broker accounts. Accounts exceeding a CER of 50% warrant immediate review; accounts exceeding 100% should be presumed excessive absent documented client instruction. For litigation counsel, the three-test structure (CER, position-level profit probability, Monte Carlo aggregate probability) provides independent, converging evidence that satisfies evidentiary standards.

For regulators, the Monte Carlo framework offers a forward-looking supplement to the traditional backward-looking CER metric. It demonstrates that fee structures are predictively harmful at inception, not merely harmful in retrospect.

SECTION9

Methodology and Citations

Legal precedents: Mihara v. Dean Witter (1980), Costello v. Oppenheimer (1986), CFTC v. Savage (1985), FINRA Rule 2111 (Suitability), BGH churning indicia. Option pricing: Black and Scholes (1973), Merton (1973). Monte Carlo: Glasserman (2004, Monte Carlo Methods in Financial Engineering, Springer). All computations use observed market data (implied volatilities at entry, realized commissions, documented trade confirmations).

The framework requires no proprietary models. Inputs are: trade confirmations, account statements, and end-of-day implied volatility surfaces from any standard data provider. All outputs are reproducible and suitable for expert testimony.

KEYEQUATION

$$CER = \text{Total Commissions} / \text{Average Account Equity}$$

Working Paper 02 — Principal Formula

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