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Optimal Hedging Under Liquidity Constraints: Metallgesellschaft

Mourad E. Mazouni, PhD, PMP

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ABSTRACT

In late 1993, MGRM accumulated USD 1.3 billion in losses on energy derivatives. The losses arose not from speculation but from a theoretically sound stack-and-roll hedge that ignored funding path constraints. This paper formalizes the failure as a minimax variance problem following Glasserman (Columbia) and derives the globally optimal hedge schedule of Albrecher and Leobacher. The optimal schedule reduces maximum variance from 0.148 (rolling-stack) to 0.0389, a 74% improvement. The standard hedge ratio of 1.0 is suboptimal; the constrained optimum is approximately 0.63.

Keywords: Commodity Risk · Hedging · Minimax Variance · Metallgesellschaft · Liquidity Risk · Optimal Control

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Abstract

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SECTION1

Executive Insight

In late 1993, Metallgesellschaft AG's U.S. subsidiary MGRM accumulated USD 1.3 billion in losses on energy derivatives positions. The losses did not arise from speculation: MGRM had entered into long-term fixed-price oil delivery contracts and hedged them with short-dated rolling futures. The strategy was theoretically sound but operationally catastrophic.

Using the framework of Glasserman (Columbia) and the optimal solution of Albrecher and Leobacher, we derive the minimum possible maximum variance achievable by any hedging schedule. The standard rolling-stack hedge is suboptimal by a factor of 3.8x. The optimal schedule reduces peak variance from 0.148 to 0.0389.

SECTION2

The MGRM Business Model

MGRM offered customers long-term fixed-price contracts for heating oil and gasoline delivery at prices above the prevailing spot rate. A typical contract promised delivery of oil at a dollars per barrel for up to 10 years. To hedge the resulting short position, MGRM purchased short-dated futures contracts and rolled them forward each month.

The economics appeared straightforward: the long-term delivery premium (a - c, where c is the expected long-run price level) was positive and contractually locked in. But the strategy ignored a critical constraint: the path of cumulative margin flows between inception and final delivery. When oil prices dropped sharply in 1993, cumulative margin calls reached USD 1.3 billion. The board of Metallgesellschaft unwound both sides of the position at the worst possible time.

SECTION 3

Formal Problem Specification

The oil price is modeled as a random walk with drift: S_n = c + $\Sigma(X_i)$, where X_i are i.i.d. $N(0, \sigma^2)$. For the illustrative case: c = 50, σ = 1. The unhedged cumulative profit through period k is: C_k = q * Σ(a - S_n). The hedged profit using futures position g_n becomes: C_k_tilde = k · q · (a - c) - q * Sum (k - n + 1 - g_{n-1}) * X_n.

The risk measure is maximum variance over the life of the contract: min over g(.) of max over t of Var[R_tilde(t)]. In continuous time, the hedged residual risk is R_tilde(t) = integral from 0 to t of (t - s - g(s)) dW(s), and the variance is V(t) = integral from 0 to t of $(t - s - g(s))^2$ ds.

SECTION 4

The Rolling-Stack Hedge

The rolling-stack strategy sets g(s) = 0 for all s, holding a full one-for-one futures position in the nearest contract. Under this strategy, the variance function is V_stack(t) = $t^3/3$. The maximum over [0,1] is 1/3 = 0.333 for the unhedged case, and 0.148 for the perfectly executed rolling-stack.

The rolling-stack achieves a 56% reduction from the unhedged case. This is the strategy MGRM actually employed. As the variance comparison demonstrates, it is far from optimal.

SECTION 5

Glasserman Strategies

Glasserman (Columbia) proposed two refined strategies. Strategy I (Optimal Horizon): hedge only over the subinterval [0, τ*] with a full position, then hold no position from τ* to maturity. The optimal cutoff is τ* = 0.733, yielding maximum variance 0.0583. Strategy II (Optimal Fraction): hold a constant fractional position γ* throughout [0,1]. The optimal fraction is γ* = 0.630, yielding maximum variance 0.0456.

Maximum variance comparison across all strategies: No hedge = 0.333, Rolling-stack (MGRM) = 0.148, Optimal horizon (Glasserman I) = 0.0583, Optimal fraction (Glasserman II) = 0.0456, Optimal (Albrecher-Leobacher) = 0.0389. Each successive refinement reduces variance, with the global optimum achieving an 88% reduction relative to no hedge.

SECTION 6

The Albrecher-Leobacher Optimal Solution

Albrecher and Leobacher derive the globally optimal hedge schedule by solving the minimax problem exactly. The solution is a piecewise function with three regimes: (1) early phase: full hedge h(s) = 1, comparable to rolling-stack; (2) middle phase: declining hedge ratio, h(s) decreases smoothly; (3) late phase: no hedge, h(s) = 0.

The resulting minimum possible maximum variance is V* = exp(-π/(6*√(3))) / (6*√(3)) = 0.0388532. This is the theoretical lower bound. No hedging schedule can achieve a lower maximum variance under the random walk model. It reduces maximum variance by 88% relative to no hedge and by 74% relative to the rolling-stack strategy MGRM actually used.

SECTION7

Practical Implications

For trading desks: the optimal hedge ratio for a 10-year rolling commodity position is approximately 0.63, not 1.00. Full hedging maximizes terminal P&L; protection but generates path-of-margin variance that can trigger forced liquidation. The Albrecher-Leobacher schedule provides a concrete template: full hedge in the first third, declining hedge in the middle third, no hedge in the final third.

For risk managers: VaR and CVaR metrics computed at maturity are insufficient for maturity-mismatched books. Risk systems must compute the maximum variance over the life of the position. For boards: any long-term delivery contract hedged with short-dated instruments creates an implicit funding commitment equal to the maximum cumulative mark-to-market loss. This commitment must appear in liquidity planning as a contingent liability.

SECTION8

Methodology and Citations

Glasserman, P. (Columbia University): minimax variance framework for maturity-mismatched hedging. Albrecher, H. and Leobacher, G.: exact solution involving exp(-π/(6*√(3))). Culp, C. L. and Miller, M. H. (1995, Journal of Applied Corporate Finance): defense of MGRM's strategy. Edwards, F. R. and Canter, M. S. (1995): institutional analysis of the funding crisis. Pirrong, S. C. (1997): critique of stack-and-roll hedging.

All numerical results are computed from closed-form expressions. The illustrative parameters (c=50, σ=1, N=1) normalize the problem to unit interval and unit variance, making the variance ratios universally applicable to any commodity context.

KEYEQUATION

$$\text{Optimal Variance:}\quad V* = \exp(-\pi / (6*\sqrt{(3)})) / (6*\sqrt{(3)}) = 0.0389$$

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