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Credit Risk Economic Capital: From Diversification Paradox to Portfolio VaR

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From Equations to Capital Research

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ABSTRACT

Credit diversification is profoundly non-linear: adding a second B-rated bond to a single A-rated bond can increase portfolio VaR at the 99.7% confidence level. Using Moody's 1920-2012 transition matrices and seniority-stratified recovery rates, this paper constructs a complete economic capital framework from first principles. A portfolio of 20 Ba-rated bonds achieves maximum loss of 3.4% at the 0.3% threshold, compared to 49.7% for a single Baa-rated bond. The Merton structural model, Vasicek one-factor model, and CDS fair spread pricing are integrated into a unified implementation framework.

Keywords: Credit Risk · Economic Capital · Value-at-Risk · Basel II/III · Merton Model · Diversification

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Abstract

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SECTION 1

Executive Insight

Credit risk modeling promises that diversification reduces portfolio loss. Yet the relationship is profoundly non-linear: adding a second B-rated bond to a single A-rated bond can increase Value-at-Risk at the 99.7% confidence level, even though expected loss falls. This paradox, documented with Moody's 1920-2012 transition data, is the central puzzle this paper resolves.

A portfolio of 20 Ba-rated bonds achieves a maximum loss of 3.4% at the 0.3% probability threshold, compared to 49.7% for a single Baa-rated bond. The economic capital requirement shrinks from 46.3% to 2.1% through diversification alone. Expected loss tells you what to provision. Unexpected loss tells you how much capital to hold. The gap between them is economic capital, and it is driven by correlation, not by default probability.

SECTION 2

The Diversification Paradox

Consider two bonds: Bond X (A-rated, PD = 33.3%, LGD = 100%) and Bond Y (B-rated, PD = 50.0%, LGD = 100%). A single Bond X has expected loss 0.333 and maximum loss 1.0. Adding Bond Y creates a portfolio with expected loss 0.833 but the probability of maximum loss is 0.333 x 0.500 = 16.7% under independence.

At the 0.3% threshold (Basel II/III standard), the single A-rated bond has zero incremental risk. But a concentrated portfolio of 2-5 B-rated names can exhibit higher tail risk than the single A-rated bond due to correlation-amplified joint default. This result drives real capital allocation decisions.

SECTION3

Moody's Transition Data: 1920-2012

The empirical foundation is Moody's historical credit transition matrix covering 92 years. Key diagonal entries (probability of remaining in the same rating class over one year): Aaa to Aaa = 86.4%, Aa to Aa = 85.8%, A to A = 86.6%, Baa to Baa = 81.4%, Ba to Ba = 73.4%, B to B = 72.3%.

For Ba-rated issuers, the 1-year default probability is 1.268%. Recovery rates by seniority from Moody's studies: Loans (secured) = 80.6%, Senior secured bonds = 63.7%, Senior unsecured = 48.6%, Subordinated = 28.5%. The spread between secured loans (80.6% recovery) and subordinated bonds (28.5%) implies that LGD assumptions alone can swing economic capital by a factor of 3.7x.

SECTION4

Expected Loss and the EL/UL Decomposition

The fundamental decomposition separates expected loss (provisioned from revenue) from unexpected loss (absorbed by capital). EL = PD x LGD x EAD. For a Baa-rated senior unsecured bond with PD = 0.189%, LGD = 51.4%, EAD = 100: EL = 0.0972.

Expected loss is a cost of doing business, priced into the credit spread. Economic capital covers the unexpected component: EC = VaR(α) - EL, where α is the confidence level (typically 99.9% for Basel II IRB, 99.7% for internal ICAAP). The challenge is computing VaR(α) for a correlated portfolio.

SECTION5

Portfolio VaR Analysis

Maximum loss at the 0.3% probability level for increasingly diversified portfolios: 1 Baa bond = 49.7%, 2 Ba bonds = 23.6%, 10 Ba bonds = 3.6%, 20 Ba bonds = 3.4%. Expected loss for all Ba portfolios is 1.3%.

A single Baa bond faces 49.7% maximum loss, essentially binary (default or no default). By the time the portfolio holds 20 Ba bonds, maximum loss converges to 3.4%, approaching the expected loss. The gap between VaR and EL, the economic capital requirement, shrinks from 46.3% to 2.1% through diversification alone. The jump from 1 to 10 names reduces tail risk by 93%; the jump from 10 to 20 reduces it by only an additional 6%.

SECTION6

Credit Portfolio Models

Three frameworks are integrated. The Merton structural model: default occurs when asset value V_T falls below the debt barrier D. $PD = \Phi(-(\ln(V0/D) + (\mu - \sigma^2/2) \cdot T) / (\sigma*\sqrt{T}))$. Distance-to-default maps equity data to credit risk through the firm's capital structure.

The Vasicek one-factor model introduces a systematic factor Z: $PD\_cond(Z) = \Phi((\Phi^{-1}(PD) - \sqrt{\rho} \cdot Z) / \sqrt{1-\rho})$, where ρ is the asset correlation. Under worst-case Z (stressed macro state), conditional PD can be orders of magnitude higher than unconditional PD. This is the mechanism by which correlation drives economic capital and the foundation of the Basel II IRB formula.

CDS fair spread pricing links market data to default probability. The fair spread s = PD x LGD x duration_adjustment. Credit Value Adjustment (CVA) extends this to bilateral counterparty risk in OTC derivatives, linking credit and market risk.

SECTION 7

Rating-Dependent Yield Curves

For credit migration from Baa to Ba, the bond is repriced at the Ba yield curve, generating a mark-to-market loss even without default. Representative credit spreads by rating: Aaa 1Y/5Y/10Y = 5/15/25 bps, Aa = 10/30/50, A = 20/55/80, Baa = 50/110/140, Ba = 150/280/340, B = 350/520/580.

A downgrade from Baa to Ba widens the 10Y spread by 200 bps, generating a mark-to-market loss of approximately 14% on a 10-year bond. This loss occurs without default and is the dominant risk mode for investment-grade credit portfolios. Migration-mode VaR is typically 2-3x larger than default-mode VaR for investment-grade books.

SECTION 8

Institutional Implications

For chief risk officers: diversification is necessary but non-linear. Recovery rate assumptions matter enormously: economic capital for a subordinated portfolio (LGD = 71.5%) is 3.7x higher than for secured loans (LGD = 19.4%). Model validation must independently stress-test recovery assumptions.

For portfolio managers: migration risk dominates default risk for investment-grade portfolios by a factor of 2-3x. Name and sector concentration limits should be calibrated from the VaR table, not notional exposure caps. For regulators: the Vasicek one-factor model captures systematic risk but not sector concentration; Pillar 2 economic capital should supplement Pillar 1 with multi-factor models.

SECTION 9

Methodology and Citations

Moody's Investors Service: historical credit transition matrices (1920-2012), recovery rate studies. Merton, R. C. (1974, Journal of Finance): structural model. Vasicek, O. (1987, 2002): one-factor Gaussian copula, ASRF model underpinning Basel II IRB. CreditMetrics (J.P. Morgan, 1997): credit migration framework. Basel Committee (2004, revised 2017): IRB approach. Schonbucher, P. (2003): CDS fair spread and CVA.

All numerical examples use Moody's published data. Portfolio VaR assumes conditional independence given the systematic factor (Vasicek model). No proprietary data or models are required.

KEY EQUATION

$$EL = PD \times LGD \times EAD; \quad EC = VaR(\square) - EL$$

Working Paper 05 — Principal Formula

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