FROMEQUATIONSTOCAPITAL · WORKINGPAPER06

Robust Portfolio Optimization with Sustainability Constraints

Mourad E. Mazouni, PhD, PMP

From Equations to Capital Research

February 2026 | Source: Volume I, Case Study IX

ABSTRACT

Markowitz mean-variance optimization is notoriously unstable: small changes in expected returns produce wildly different portfolios. This paper integrates sustainability constraints with robust optimization using a 5-product universe, 500,000 Monte Carlo portfolio draws, and three robust methods (ellipsoidal uncertainty, James-Stein shrinkage, worst-case mean). Imposing sustainability threshold $\tau \geq 0.55$ reduces maximum Sharpe from 1.455 to 1.215, a 16.5% cost. But the sustainability-constrained robust portfolio shows zero out-of-sample degradation (1.12 in-sample and out-of-sample), while nominal Markowitz collapses from 1.455 to 0.82.

Keywords: Portfolio Construction · Robust Optimization · Sustainability · ESG · Sharpe Ratio · Monte Carlo

Table of Contents

Abstract

Markowitz mean-variance optimization is notoriously unstable: small changes in expected returns produce wildly different portfolios. This paper integrates sustainability constraints with robust optimization using a 5-product universe, 500,000 Monte Carlo portfolio draws, and three robust methods (ellipsoidal uncertainty, James-Stein shrinkage, worst-case mean). Imposing sustainability threshold $\tau \geq 0.55$ reduces maximum Sharpe from 1.455 to 1.215, a 16.5% cost. But the sustainability-constrained robust portfolio shows zero out-of-sample degradation (1.12 in-sample and out-of-sample), while nominal Markowitz collapses from 1.455 to 0.82.

SECTION1

Executive Insight

Standard mean-variance optimization is an estimation-error maximizer: it produces extreme allocations that concentrate in assets with the largest upward estimation errors. This paper integrates sustainability constraints with robust optimization to produce portfolios that are simultaneously efficient, stable, and ESG-compliant.

Using a 5-product universe, imposing a sustainability threshold of $\tau \geq 0.55$ reduces the maximum achievable Sharpe ratio from 1.455 to 1.215, a 16.5% cost. But when parameter uncertainty is accounted for through robust optimization, the sustainability-constrained portfolio outperforms the nominal unconstrained portfolio in 68% of out-of-sample scenarios. The sustainability constraint acts as an implicit regularizer, shrinking the feasible set in dimensions most vulnerable to estimation error.

SECTION2

Markowitz with Sustainability Constraints

The classical Markowitz problem maximizes portfolio return for a given risk level. The sustainability extension adds a linear constraint: $\tau'w \geq \tau_0$, where τ_i is the sustainability score of asset i and τ_0 is the minimum requirement.

The 5-product universe: Product 1 (return 3.0%, vol 2.0%, $\tau = 0.20$), Product 2 (4.5%, 3.5%, 0.35), Product 3 (5.0%, 4.0%, 0.50), Product 4 (6.0%, 5.5%, 0.55), Product 5 (7.0%, 7.0%, 0.70). Portfolio metrics are: $\mu\_P = \Sigma(w_i * \mu_i)$, $\sigma\_P = \sqrt{(w' * \Sigma * w)}$, $\tau\_P = \Sigma(w_i * \tau_i)$.

SECTION 3

The Sustainability-Sharpe Trade-off

The Normalized Sustainability Ratio (NSR) embeds the sustainability score directly into the performance metric: NSR(γ) = τ_P^γ * SR_P. When γ = 0, NSR reduces to the classical Sharpe ratio. As γ increases, portfolios with higher sustainability scores are progressively favored.

Monte Carlo simulation with 500,000 random portfolios per scenario maps the trade-off frontier. Maximum Sharpe under sustainability constraints: unconstrained = 1.455, τ ≥ 0.55 yields 1.215 (16.5% cost), τ ≥ 0.60 yields 1.055 (27.5% cost), τ ≥ 0.68 yields 0.795 (45.4% cost). The cost is non-linear: the first 0.15 units of sustainability improvement costs only 16.5% of Sharpe, while the next 0.13 units costs an additional 28.9%.

SECTION 4

Analytical Solutions: Lagrange Method

The sustainability-constrained Markowitz problem admits closed-form solution via Lagrange multipliers. The augmented objective includes three constraints: target return, full investment, and minimum sustainability. The first-order conditions yield: $w^* = (1/2) * \Sigma^{-1} * (\lambda_1 * \mu + \lambda_2 * 1 + \lambda_3 * \tau)$.

The three multipliers satisfy a 3x3 linear system involving $\mu' * \Sigma^{-1} * \mu$, $\mu' * \Sigma^{-1} * 1$, $\mu' * \Sigma^{-1} * \tau$, and their permutations. The solution is exact and computationally trivial. This analytical solution provides the nominal optimum; the robust extensions protect against the estimation errors that make this nominal solution unreliable in practice.

SECTION 5

Robust Optimization Framework

Three robust approaches address parameter fragility. Ellipsoidal Uncertainty Sets: the true mean is assumed to lie within an ellipsoid centered on the estimate. The robust problem reduces to: max w'*μ_hat - κ*√(w'*Σ · w) subject to full investment. The penalty term automatically penalizes concentration.

James-Stein Shrinkage: blends the sample mean toward a structured target (typically the global minimum-variance return). The shrinkage intensity $\alpha = (n-2)/T / \|\mu\_hat - \mu_0\|^2$ is provably superior to the sample mean for n ≥ 3, the Stein paradox applied to portfolio construction.

Worst-Case Mean (Palomar, 2025): computes the worst-case expected return directly, separating estimation risk from market risk. μ_worst(w) = μ_hat'*w - κ*√(w'*Σ_mu · w), where Σ_mu is the covariance of the mean estimator.

SECTION 6

Numerical Results: Monte Carlo Validation

All three methods are validated against the nominal Markowitz solution using 500,000 random portfolio draws per scenario. Key results by method:

Nominal Markowitz: in-sample SR = 1.455, out-of-sample SR = 0.82, max weight = 68%, quarterly turnover = 34%. Ellipsoidal robust (κ=1): 1.31 / 1.05 / 31% / 14%. James-Stein shrinkage: 1.28 / 1.09 / 28% / 12%. Sustainability + robust (τ ≥ 0.55, κ=1): 1.12 / 1.12 / 25% / 9%.

The nominal portfolio collapses 44% out-of-sample. The sustainability-constrained robust portfolio shows zero degradation (1.12 both in-sample and out-of-sample) because the sustainability constraint eliminates concentrated positions most vulnerable to estimation error. Maximum single-asset weight falls from 68% to 25%.

SECTION7

Institutional Implications

For investment committees: ESG constraints should not be viewed as pure performance drag. Their portfolio-level effect is equivalent to shrinkage toward a diversified prior, reducing estimation sensitivity and improving out-of-sample stability. The Sustainability-Sharpe frontier provides a quantitative tool for choosing the ESG ambition level.

For quant portfolio managers: the gap between in-sample and out-of-sample Sharpe is 0.64 for nominal Markowitz but 0.00 for the robust + sustainability variant. Parameter uncertainty, not model specification, is the binding constraint. For regulators: the Sustainability-Sharpe frontier provides a standardized, quantitative disclosure format for cross-fund ESG cost comparison.

SECTION8

Methodology and Citations

Markowitz, H. (1952, Journal of Finance): mean-variance framework. Palomar, D. P. (2025, Portfolio Optimization: Theory and Application): worst-case mean, ellipsoidal sets. Ben-Tal and Nemirovski (1999, Mathematics of Operations Research): robust convex optimization. Goldfarb and Iyengar (2003, Mathematical Programming): robust portfolio selection. James and Stein (1961): inadmissibility of the sample mean. Black and Litterman (1992, Financial Analysts Journal): Bayesian views. Del Chicca and Mazouni: sustainability-constrained Sharpe ratio.

All results use the 5-product universe. Monte Carlo draws use 500,000 random weight vectors per scenario with full-investment and long-only constraints.

KEYEQUATION

```
NSR(▯) = ▯^▯ * SR = ▯^▯ * (▯ - rf) / ▯
```

Working Paper 06 — Principal Formula

Disclaimer. This working paper is shared for educational and research purposes only. It does not constitute investment advice, a solicitation to buy or sell securities, or an offer of financial services. All quantitative examples are illustrative and based on hypothetical or historical data. Past performance is not indicative of future results. The author and publisher expressly disclaim all liability for investment decisions made on the basis of information contained herein. Please consult qualified professionals for specific investment decisions.

Source. Derived from the From Equations to Capital two-volume research program — a professional system bridging quantitative methods and capital markets. For the complete research library, visit equationstocapital.com.

Σ → $

From Equations to Capital

A Two-Volume Professional System

The definitive professional system bridging quantitative methods and capital markets. 27 chapters spanning corporate finance, derivatives, risk management, and portfolio optimization.

equationstocapital.com

Working Paper 06 · February 2026